Name ______________________ Student Number ______________

All solutions are to be presented on the paper in the space provided. The quiz is open book. You can discuss the problem with others and ask the TA questions.

(1) Let $f(x) = x^2$ and $g(x) = 2x^2$. Call the linearization of $f(x)$ $L_f$ and the linearization of $g(x)$ $L_g$. Answer the following questions:

(a) Find $L_f(x)$ and $L_g(x)$

The linearization of a function at a point $x = a$ is $L(x) = f(a) + f'(a)(x-a)$, so

$$L_f(x) = a^2 + 2a(x-a)$$
$$L_g(x) = 2a^2 + 4a(x-a)$$

(b) Use $L_f$ and $L_g$ to estimate $f(x)$ and $g(x)$ at $x = 1.1$

Use $a = 1$, since it's close to 1.1 and is easy to compute with. Then,

$$L_f(x) = 1 + 2(x-1)$$
$$L_g(x) = 2 + 4(x-1)$$

So that

$$L_f(1.1) = 1 + 2(1.1-1) = 1.2$$
$$L_g(1.1) = 2 + 4(1.1-1) = 2.4$$

Over→

(c) Using the exact values of $f$ and $g$ at $x = 1.1$, find the error in the estimation. That is, calculate $|L_f(1.1) - f(1.1)|$ and $|L_g(1.1) - g(1.1)|$

$$f(1.1) = 1.21$$
$$g(1.1) = 2.42$$

So that

$$|L_f(1.1) - f(1.1)| = 0.01$$
$$|L_g(1.1) - g(1.1)| = 0.02$$

(d) Explain why the error in the linearization of $g$ is twice as large as the error in the linearization of $f$.
The graph of $g$ moves away from its tangent line at $x = 1$ twice as fast as the graph of $f$. How fast a graph bends is measured by its second derivative. Notice that $g''(x) = 4$ and $f''(x) = 2$.

(2) Give examples of functions that satisfy each criteria below. Note that each item refers to a different function.
(a) A function that is continuous on $(a, b)$ but does not have a maximimum.
$f(x) = \frac{1}{x}$ on $(0, 1)$

Over→

(b) A function that is defined on a closed interval $[a, b]$, but does not have a maximum.

$$f(x) = \begin{cases} x, & 0 \le x < 1 \\ x - 1, & 1 \le x \le 1.5 \end{cases}$$

(3) Let $f(x) = x \sin x$ defined on $[0, \frac{\pi}{2}]$. State the theorem that guarantees the existence of a maximum and minimum for $f(x)$ and find the absolute maximum and minimum.
The extreme value theorem: if $f(x)$ is continuous on $[a.b]$ then $f(x)$ has a maximum and a minimum in $[a, b]$. $f(x) = x\sin(x)$ is continuous everywhere, so the extreme value theorem applies.

$$f'(x) = \sin(x) + x\cos(x)$$

Solve $f'(x) = 0$,

$$\sin(x) + x\cos(x) = 0$$
$$x = -\tan(x)$$

This has $x = 0$ as the only solution in $[0, \frac{\pi}{2}]$, so the extreme values can only occur at the endpoints.

$$f(0) = 0 \qquad \text{minimum}$$
$$f(\frac{\pi}{2}) = \frac{\pi}{2} \qquad \text{maximum}$$